

Einstein Dynamics as Admissibility Balance

A Collapse-Comonadic Reconstruction of Geometric Physics

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April 17, 2026

Abstract

We present a structural reformulation of geometric physics in which connection, curvature, and Einstein dynamics arise from collapse-selection processes acting on relational configuration space. Working in a bicategorical setting equipped with a lax idempotent comonad, we interpret admissible structure as coalgebraic stability under collapse and define transport as a collapse-compatible lifting into this stable sector.

Within this framework, geodesics are identified with horizontal coalgebraic flows, connections with lifting rules preserving admissibility, and curvature with the second-order failure of lifting to compose coherently. A distinguished connection emerges as the unique admissibility-compatible symmetric lifting minimizing a tension functional, recovering the Levi-Civita connection in the metric-closure regime.

We further show that the Einstein field equations arise as a balance condition between curvature (a lifting defect) and a stress term given by variation of admissibility tension. In the appropriate limit, this formulation reproduces classical General Relativity, including Schwarzschild geometry and the Newtonian potential, while naturally predicting deviations in regimes where admissibility structure fails to admit a metric representation.

The result provides a categorical and selection-based reinterpretation of geometric physics in which classical spacetime structure appears as an emergent coherence condition rather than a primitive ontology.

1 Introduction

General Relativity (GR) provides an extraordinarily successful description of gravitational phenomena through the geometry of spacetime. In this formulation, connection, curvature, and geodesic structure are taken as fundamental, and the Einstein field equations relate curvature to stress-energy.

Despite this success, the geometric formulation leaves open a structural question: why should physical dynamics be governed by geometric consistency conditions at all? In particular, GR presupposes the existence of a smooth metric structure without explaining the mechanism by which such structure arises or when it may fail.

Across physics and mathematics, a recurring pattern appears: a space of possible configurations is constrained, unstable configurations are suppressed, and a restricted set of stable structures persists. This pattern underlies renormalization, decoherence, thermodynamics, and categorical formulations of process structure.

In this work, we take this pattern as fundamental and develop a framework in which geometric structure arises from selection under constraint. We formalize this using a collapse-selection operator acting on relational configuration space, inducing a lax idempotent comonad whose coalgebras represent admissible, stable structure.

Within this setting, we reinterpret core geometric notions:

- Geodesics as collapse-stable (coalgebraic) trajectories,

- Connections as collapse-compatible liftings,
- Curvature as the failure of lifting to compose coherently,
- Einstein dynamics as a balance between curvature and admissibility tension.

We show that in a metric-closure regime, this framework reproduces the standard structures of GR, including the Levi–Civita connection, Schwarzschild geometry, and the Newtonian limit. Outside this regime, it predicts controlled deviations arising from the breakdown of global admissibility coherence.

The goal of this paper is not to replace GR, but to provide a structural reconstruction in which geometry appears as an emergent consistency condition of admissibility-preserving transport.

2 Collapse-Selection and Admissible Structure

Let Σ denote a relational configuration space equipped with a collapse operator

$$\Phi : \Sigma \rightarrow \Sigma,$$

and define admissible structure as the invariant sector

$$\text{Fix}(\Phi) = \{x \in \Sigma \mid \Phi(x) = x\}.$$

This induces a lax idempotent comonad

$$\text{Coll} : \mathcal{C} \rightarrow \mathcal{C}$$

on a bicategory \mathcal{C} of relational configurations, whose coalgebras represent collapse-stable structure.

3 Collapse-Stable Trajectories as Generalized Geodesics

In classical geometry, geodesics are defined as trajectories that extremize an action functional or, equivalently, as autoparallel curves under a connection. These formulations presuppose a background geometric structure from which admissible directions and variation are defined.

In Quantum Collapse Geometry (QCG), geometry is not primitive. Instead, admissibility and persistence are determined by collapse-selection dynamics acting on a relational configuration space. We show that classical geodesics arise as a special case of a more general notion: collapse-stable trajectories.

Definition (Generalized Geodesic). Let Σ be a relational configuration space equipped with a collapse operator

$$\Phi : \Sigma \rightarrow \Sigma,$$

and let

$$P_\lambda : \Sigma \rightarrow S_\lambda$$

be a coarse-graining projection to a scale- λ descriptive space inducing an effective generator G_λ .

A trajectory $\gamma_\lambda(t) \in S_\lambda$ is a *generalized geodesic* if:

1. (Admissibility) γ_λ lies in the admissible sector induced by Φ ,
2. (Local indistinguishability) admissible variations $\delta\gamma_\lambda$ do not produce a first-order distinguishable change in the action functional,
3. (Persistence) γ_λ remains within a collapse-stable sector, equivalently lying in the persistent (near-fixed-point) structure of G_λ .

Proposition (Collapse-Stable Trajectories as Generalized Geodesics). Let $\Phi : \Sigma \rightarrow \Sigma$ be a collapse-selection operator and $\{G_\lambda\}$ the induced scale-local generators under coarse-graining. Then any trajectory γ_λ that is stable under the induced collapse dynamics—i.e., lies within an invariant or persistent sector of G_λ —is a generalized geodesic of the admissibility structure.

Moreover, if the admissible sector admits an effective geometric representation (metric or connection), γ_λ reduces to a classical geodesic.

Proof Sketch.

- Collapse-selection dynamics iteratively eliminate inadmissible configurations, restricting Σ to an invariant sector $\text{Fix}(\Phi)$ or its basin of attraction.
- Under projection P_λ , this induces an effective dynamics G_λ on S_λ encoding persistence structure rather than generative evolution.
- By the Lagrangian reinterpretation, physical trajectories are those locally indistinguishable under admissible variations; i.e., stationary under the induced distinguishability ordering.
- Stability under G_λ implies that admissible perturbations are suppressed, so the trajectory remains invariant under collapse-induced selection.
- Therefore, γ_λ satisfies both:

$$\delta S_{\text{adm}}[\gamma_\lambda] = 0, \quad \gamma_\lambda \in \text{Pers}(G_\lambda),$$

establishing it as a generalized geodesic.

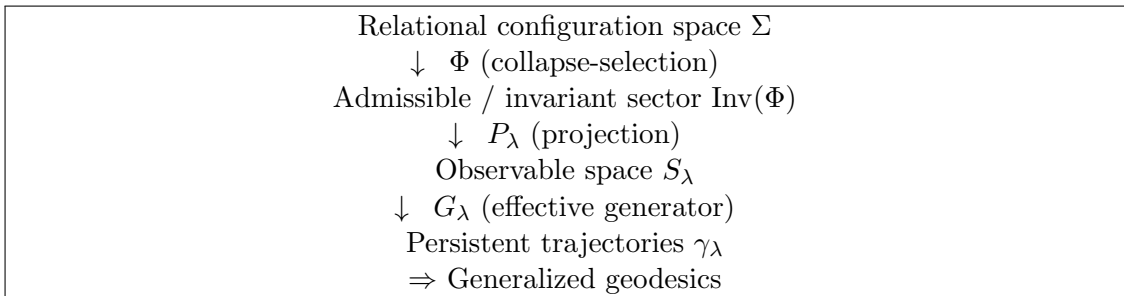
- When the admissible sector closes under an effective geometric structure, this condition reduces to the classical geodesic equation.

Interpretation. In this formulation:

- Classical geodesics are not fundamental, but emerge as fixed-point trajectories under collapse-ordered operations. [1]
- The action functional orders trajectories by distinguishability in degree-of-freedom space, selecting those invariant under admissible variation. [2]
- Collapse acts as a selection operator restricting dynamics to admissible sectors, with stable configurations corresponding to invariant structure. [3]

Thus, geodesic motion is reinterpreted as the observable image of persistence under collapse-selection, rather than as a primitive geometric law.

Diagram (Structure of Generalized Geodesics).



Remark. This formulation unifies:

- variational principles (stationarity under admissible variation),
- dynamical stability (persistence under G_λ),
- and collapse-selection (invariance under Φ),

into a single structural condition. Geometry appears only when this admissibility structure admits a metric representation, and breaks down when it does not.

3.1 Geodesics as Coalgebras of the Collapse Comonad

We refine the notion of generalized geodesics by expressing collapse-stable trajectories as coalgebras of the collapse comonad induced by admissible dynamics.

Setting. Let \mathcal{C} be a bicategory of relational configurations, and let

$$\text{Coll} : \mathcal{C} \rightarrow \mathcal{C}$$

be a lax idempotent comonad induced by admissible collapse dynamics, with counit $\varepsilon : \text{Coll} \Rightarrow \text{Id}$ and comultiplication $\delta : \text{Coll} \Rightarrow \text{Coll} \circ \text{Coll}$.

Stable configurations correspond to coalgebras of Coll , forming a coreflective subcategory $\mathcal{C}_{\text{stable}} \subset \mathcal{C}$.

Definition (Geodesic Coalgebra). Let $X \in \mathcal{C}$ be a relational configuration object. A *geodesic coalgebra* is a coalgebra

$$\gamma : X \rightarrow \text{Coll}(X)$$

such that:

1. (Stability) γ satisfies the coalgebra axioms up to coherent 2-cells,

$$(\text{Coll}(\gamma) \circ \gamma) \simeq (\delta_X \circ \gamma), \quad (\varepsilon_X \circ \gamma) \simeq \text{id}_X,$$

2. (Local minimality) the induced trajectory under γ is locally invariant under admissible perturbations,
3. (Persistence flow) the image of γ lies within a persistent sector of the induced scale-local generator.

Proposition (Geodesics as Collapse Coalgebras). Let Coll be a lax idempotent comonad induced by admissible dynamics. Then:

1. Coalgebras of Coll correspond to collapse-stable configurations,
2. Paths internal to the coalgebra category $\mathcal{C}_{\text{stable}}$ correspond to persistence-preserving trajectories,
3. Such trajectories define generalized geodesics of the admissibility structure.

In particular, classical geodesics arise as those coalgebraic trajectories for which the induced admissibility structure admits a metric representation.

Proof Sketch.

- By construction, Coll is idempotent up to coherence:

$$\text{Coll}^2 \simeq \text{Coll},$$

so its coalgebras define stable objects. [4]

- A coalgebra $\gamma : X \rightarrow \text{Coll}(X)$ identifies X with its collapse-stabilized image, ensuring invariance under admissible dynamics.
- Internal morphisms in $\mathcal{C}_{\text{stable}}$ preserve this structure, so trajectories within this subcategory remain collapse-stable.
- Under projection to an observable scale, these trajectories correspond to paths that are locally indistinguishable under admissible variation, i.e., stationary under the induced action functional.
- Therefore, such trajectories satisfy the definition of generalized geodesics.

Relation to KZ Doctrines. This structure is formally dual to Kock–Zöberlein (KZ) doctrines:

- KZ doctrines arise from lax idempotent monads T , where stable objects are T -algebras closed under free completion,
- Collapse-selection induces a lax idempotent comonad Coll , where stable objects are coalgebras selected under admissible dynamics,
- The direction of structure is reversed:

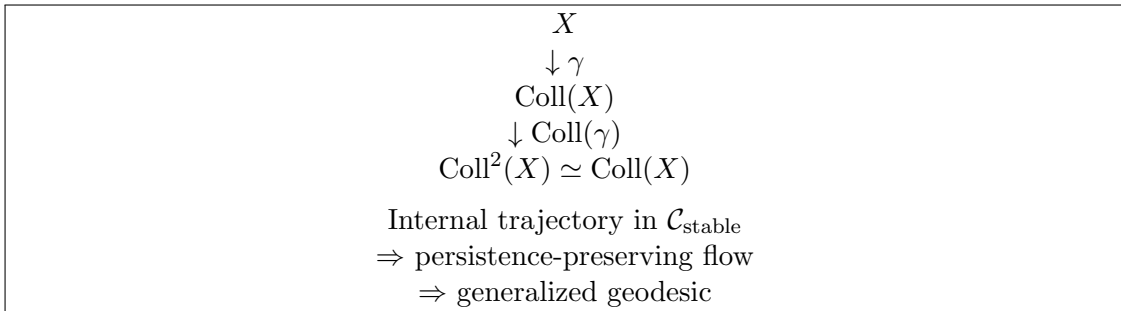
$$\text{KZ: generation } (X \rightarrow T(X)), \quad \text{QCG: selection } (X \rightarrow \text{Coll}(X)).$$

Thus:

KZ Doctrine: stability via closure under generation
QCG Collapse Doctrine: stability via invariance under selection

Geodesics, in this setting, are not primitive geometric objects but arise as coalgebraic flows within the coreflective subcategory of collapse-stable structure.

Diagram (Coalgebraic Geodesic Structure).



Interpretation. This formulation yields a structural reclassification:

- Geodesics are not defined by a metric, but by coalgebraic stability,
- Geometry appears only when the collapse-stable sector admits a metric representation,
- Outside such regimes, geodesics persist as admissibility-preserving flows without geometric interpretation.

4 Connection as Collapse-Compatible Lifting

We extend the coalgebraic formulation of generalized geodesics by identifying the notion of a connection with a choice of lifting compatible with the collapse comonad.

Setting. Let \mathcal{C} be a bicategory of relational configurations, and let

$$\text{Coll} : \mathcal{C} \rightarrow \mathcal{C}$$

be a lax idempotent comonad induced by admissible collapse dynamics, with coreflective inclusion

$$i : \mathcal{C}_{\text{stable}} \hookrightarrow \mathcal{C}, \quad i \dashv \text{Coll}.$$

Let

$$P_\lambda : \mathcal{C} \rightarrow \mathcal{C}_\lambda$$

be a projection to an observable (coarse-grained) category.

Definition (Collapse-Compatible Lifting). A *collapse-compatible lifting* is a rule assigning to each morphism

$$f : X \rightarrow Y \quad \text{in } \mathcal{C}_\lambda$$

a lift

$$\tilde{f} : \tilde{X} \rightarrow \tilde{Y} \quad \text{in } \mathcal{C}_{\text{stable}}$$

such that the following diagram commutes up to coherent 2-cells:

$$\begin{array}{ccc} \tilde{X} & \xrightarrow{\tilde{f}} & \tilde{Y} \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & Y \end{array}$$

and the lifting is compatible with collapse in the sense that

$$\Delta_f := \tilde{f} \circ \gamma_X \Rightarrow \gamma_Y \circ f,$$

where $\gamma_X : X \rightarrow \text{Coll}(X)$ and $\gamma_Y : Y \rightarrow \text{Coll}(Y)$ are the associated coalgebra structures, and

$$\|\Delta_f\|^2$$

is the size of the comparison 2-cell.

The difference is understood after projection to a linearized observable representation where such norms are defined.

Definition (Connection). A *connection* on \mathcal{C}_λ is a choice of collapse-compatible lifting for all morphisms, functorial up to coherent 2-cells.

Equivalently, a connection is a section (up to coherence) of the projection

$$P_\lambda : \mathcal{C}_{\text{stable}} \rightarrow \mathcal{C}_\lambda$$

that preserves coalgebra structure.

Proposition (Geodesics as Horizontal Coalgebraic Flows). Let a connection be given by collapse-compatible lifting.

Then:

1. A trajectory γ_λ in \mathcal{C}_λ is a geodesic if and only if its lift $\tilde{\gamma}$ is a morphism internal to $\mathcal{C}_{\text{stable}}$,
2. Such trajectories are precisely those whose evolution is entirely contained within collapse-stable structure,
3. The connection defines a notion of horizontality as preservation of coalgebraic structure under lifting.

Proof Sketch.

- By construction, $\mathcal{C}_{\text{stable}}$ consists of coalgebras of the collapse comonad, representing admissible invariant structure.
- A lifting \tilde{f} preserves admissibility if it commutes with the coalgebra structure.
- A trajectory is horizontal if its lift remains entirely within $\mathcal{C}_{\text{stable}}$, i.e., does not leave the admissible sector.
- Under projection, such trajectories correspond to paths locally invariant under admissible perturbations.
- Therefore, horizontal lifts define generalized geodesics.

Interpretation. This formulation reinterprets connection and curvature structurally:

- A connection is not a primitive geometric object, but a rule for lifting observable structure into collapse-stable structure,
- Horizontality corresponds to preservation of admissibility under evolution,
- Geodesics arise as horizontal coalgebraic flows.

Curvature (Failure of Global Collapse Compatibility). Given a connection defined by collapse-compatible lifting, curvature is measured by the failure of lifting to be path-independent.

For composable morphisms f, g , curvature is detected by the deviation:

$$\widetilde{g \circ f} \neq \tilde{g} \circ \tilde{f}.$$

Interpretation:

- Zero curvature corresponds to global compatibility of admissibility structure,
- Nonzero curvature reflects obstruction to extending collapse-stable structure consistently across the observable category,
- Geometric curvature emerges as the observable signature of this obstruction.

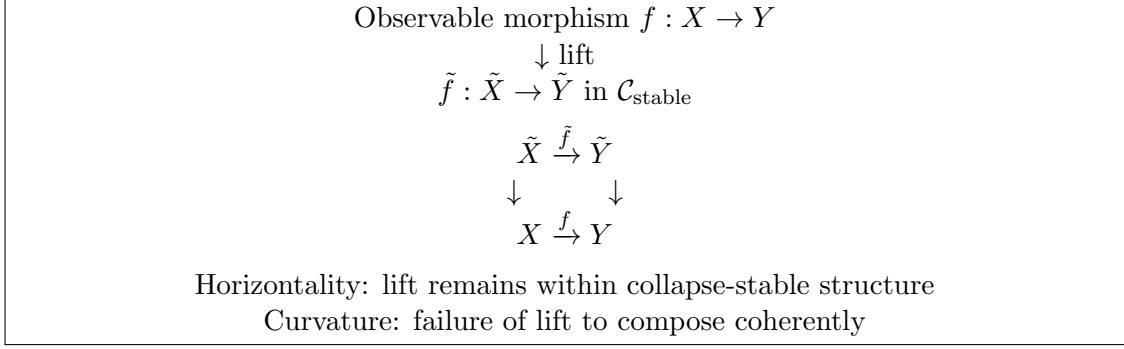
Relation to KZ Doctrines and Selection-Induced Structure. In KZ doctrine theory, a connection-like structure arises from lifting along a monadic completion.

Here, the dual structure appears:

- KZ: lift into freely generated structure (closure),
- QCG: lift into collapse-stable structure (selection).

Thus, a connection in QCG is a choice of lifting compatible with a selection-induced doctrine, rather than a completion doctrine.

Diagram (Connection as Collapse-Compatible Lifting).



5 Levi–Civita Connection as Minimal Admissibility Lifting

We now refine the notion of connection as collapse-compatible lifting by identifying a distinguished class of such liftings selected by a variational minimality principle. This plays the role, within Quantum Collapse Geometry (QCG), of the classical Levi–Civita connection.

Setting. Let \mathcal{C} be a bicategory of relational configurations, and let

$$\text{Coll} : \mathcal{C} \rightarrow \mathcal{C}$$

be a lax idempotent comonad induced by admissible collapse dynamics, with

$$i : \mathcal{C}_{\text{stable}} \hookrightarrow \mathcal{C}, \quad i \dashv \text{Coll}.$$

Let

$$P_\lambda : \mathcal{C}_{\text{stable}} \rightarrow \mathcal{C}_\lambda$$

denote the projection to a scale- λ observable category.

A connection is a choice of collapse-compatible lifting of morphisms in \mathcal{C}_λ into $\mathcal{C}_{\text{stable}}$.

Admissibility Tension. Let ∇ be such a lifting rule. We define its admissibility tension by a functional

$$\mathbb{T}[\nabla] : \text{Conn}(\mathcal{C}_\lambda) \rightarrow \mathbb{R}_{\geq 0},$$

assigning to each collapse-compatible lifting a non-negative quantity measuring the aggregate distinguishability introduced by transport under ∇ relative to the collapse-stable structure.

In particular, \mathbb{T} is required only to be well-defined up to coherent equivalence of liftings and to be monotone under admissibility-preserving refinement.

Formally, $\mathbb{T}[\nabla]$ is required to satisfy:

1. $\mathbb{T}[\nabla] \geq 0$,
2. $\mathbb{T}[\nabla] = 0$ precisely when ∇ preserves admissible relational structure without introducing additional distinguishable discrepancy,
3. $\mathbb{T}[\nabla]$ is functorial up to coherent 2-cells under collapse-compatible composition.

This functional is the connection-level analogue of the distinguishability ordering induced by the action on trajectories.

Canonical Example (Quadratic Admissibility Tension). A natural class of admissibility tension functionals is obtained by measuring the failure of a lifting to preserve coalgebraic structure.

Let ∇ be a collapse-compatible lifting assigning to each morphism $f : X \rightarrow Y$ a lift $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$. Define the local admissibility defect:

$$\Delta_f := \tilde{f} \circ \gamma_X - \gamma_Y \circ f,$$

where $\gamma_X : X \rightarrow \text{Coll}(X)$ and $\gamma_Y : Y \rightarrow \text{Coll}(Y)$ are the associated coalgebra structures.

We then define the admissibility tension by a quadratic functional:

$$\mathbb{T}[\nabla] := \sum_{f \in \mathcal{G}} \|\Delta_f\|^2,$$

where:

- \mathcal{G} is a generating family of morphisms (e.g., local transports),
- $\|\cdot\|$ denotes a norm induced after projection to observable structure.

Interpretation. In this form, $\mathbb{T}[\nabla]$ measures the total squared failure of collapse-compatible lifting to preserve admissibility across local transport.

- $\mathbb{T}[\nabla] = 0$ if and only if the lifting is strictly admissibility-compatible,
- small \mathbb{T} corresponds to near-coherent admissibility structure,
- minimization of \mathbb{T} selects the least distinguishability-inducing transport.

Toy Realization: From Admissibility Tension to an Effective Stress Tensor. To make the stress-admissibility tensor more explicit, consider a coarse-grained scalar field

$$\psi : M \rightarrow \mathbb{R}$$

on an emergent metric background (M, g) , interpreted as a local admissibility-burden field. We take a minimal quadratic admissibility-tension functional of Dirichlet type:

$$\mathbb{T}[\psi; g] = \int_M \left(\frac{\kappa}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + V_{\text{adm}}(\psi) \right) \sqrt{-g} d^4x,$$

where:

- $\kappa > 0$ sets the stiffness of admissibility variation,
- $V_{\text{adm}}(\psi)$ is an effective admissibility potential.

This functional measures the cost of spatial and temporal variation in admissibility burden together with the local cost of maintaining a given admissibility profile.

Definition (Toy Stress-Admissibility Tensor). Define the effective stress-admissibility tensor by metric variation:

$$S_{\mu\nu}^{(\psi)} := -\frac{2}{\sqrt{-g}} \frac{\delta \mathbb{T}}{\delta g^{\mu\nu}}.$$

A direct computation gives

$$S_{\mu\nu}^{(\psi)} = \kappa \partial_\mu \psi \partial_\nu \psi - g_{\mu\nu} \left(\frac{\kappa}{2} g^{\alpha\beta} \partial_\alpha \psi \partial_\beta \psi + V_{\text{adm}}(\psi) \right).$$

Interpretation. This has exactly the form of a scalar-field stress tensor, but here it is interpreted structurally rather than ontologically:

- ψ does not represent a fundamental matter field,
- it represents a coarse-grained profile of admissibility tension,
- $S_{\mu\nu}^{(\psi)}$ measures how non-uniform admissibility burden distorts the effective lifting geometry.

Thus, the standard stress-energy form is recovered as the metric response of a quadratic admissibility-tension functional.

Resulting Balance Law. Substituting this toy realization into the admissibility balance equation yields

$$\text{Ric}_{\text{Coll}\mu\nu} - \frac{1}{2}\mathcal{R}_{\text{Coll}}g_{\mu\nu} = \lambda S_{\mu\nu}^{(\psi)}.$$

In the metric-closure regime, where

$$\text{Ric}_{\text{Coll}\mu\nu} \rightarrow R_{\mu\nu}, \quad \mathcal{R}_{\text{Coll}} \rightarrow R,$$

this becomes

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \lambda \left[\kappa \partial_\mu \psi \partial_\nu \psi - g_{\mu\nu} \left(\frac{\kappa}{2} g^{\alpha\beta} \partial_\alpha \psi \partial_\beta \psi + V_{\text{adm}}(\psi) \right) \right].$$

Static Weak-Field Limit. If ψ is static and slowly varying, then the dominant contribution is the effective admissibility density

$$\rho_{\text{adm}} \approx \frac{\kappa}{2} |\nabla \psi|^2 + V_{\text{adm}}(\psi),$$

and the 00-component of the balance law reduces in the weak-field limit to a Poisson-type equation

$$\nabla^2 \phi \propto \rho_{\text{adm}},$$

recovering the interpretation of the Newtonian potential as sourced by admissibility tension.

Remark. This construction is intended only as a canonical toy model. Its role is not to identify the final microscopic form of admissibility tension, but to show explicitly how a quadratic tension functional induces a $T_{\mu\nu}$ -like source term by standard variational means. More refined realizations may be built from vector-, tensor-, or categorical defect fields, but the essential structural point is already visible in the scalar case.

Definition (Admissibility-Compatible Connection). A collapse-compatible lifting ∇ is *admissibility-compatible* if transport preserves the local admissibility structure selected by Coll .

Equivalently, for each admissible object X and lifted morphism

$$\tilde{f} : \tilde{X} \rightarrow \tilde{Y},$$

the coalgebraic structure is preserved up to coherent 2-cell:

$$\tilde{f} \circ \gamma_X \simeq \gamma_Y \circ f.$$

This is the QCG analogue of metric compatibility: the connection preserves the distinguishability/persistence structure that defines the local admissible sector.

Definition (Symmetric Collapse Lifting). A collapse-compatible lifting ∇ is *symmetric* if infinitesimal admissible transport is order-independent up to coherent equivalence.

That is, for admissible local directions u, v , the induced lifted transport satisfies

$$\nabla_u v \simeq \nabla_v u$$

up to the canonical comparison 2-cell, so that no residual order-bias is introduced by the lifting itself.

This is the QCG analogue of torsion-freeness: the lifting does not encode spurious asymmetry beyond that already present in the admissible relational structure.

Definition (Levi–Civita Collapse Connection). A *Levi–Civita collapse connection* is a collapse-compatible lifting ∇^{LC} satisfying:

1. admissibility compatibility,
2. symmetry,
3. minimal admissibility tension:

$$\mathbb{T}[\nabla^{LC}] \leq \mathbb{T}[\nabla]$$

for all admissibility-compatible symmetric liftings ∇ .

Proposition (Levi–Civita as Unique Minimal Collapse Lifting). Suppose the observable sector \mathcal{C}_λ admits a sufficiently regular local admissibility structure, and suppose the admissibility tension functional $\mathbb{T}[\nabla]$ is strictly convex on the class of admissibility-compatible symmetric liftings.

Then there exists a unique collapse-compatible lifting

$$\nabla^{LC}$$

minimizing admissibility tension. This lifting defines the QCG analogue of the Levi–Civita connection.

Proof Sketch.

- Collapse-selection defines the locally admissible relational sector; coalgebraic structure identifies the stable subcategory.
- Admissibility-compatible liftings are precisely those preserving this selected structure under transport.
- Symmetry removes order-dependent artifacts of lifting, leaving only structure forced by the admissible sector itself.
- The functional $\mathbb{T}[\nabla]$ measures the distinguishability cost induced by transport. By the degree-of-freedom interpretation of action, minimality of such distinguishability selects the dynamically preferred structure.
- Strict convexity implies existence and uniqueness of the minimizer ∇^{LC} .
- Hence ∇^{LC} is the unique admissibility-compatible symmetric lifting whose transport introduces the least additional admissibility tension.

Geodesic Equation. A trajectory in the observable sector is a geodesic relative to ∇^{LC} precisely when its horizontal lift remains internally coalgebraic and extremizes no additional admissibility tension.

Equivalently, geodesics are the trajectories whose transport is already optimal with respect to the collapse-stable relational structure.

Interpretation. This yields the following correspondence:

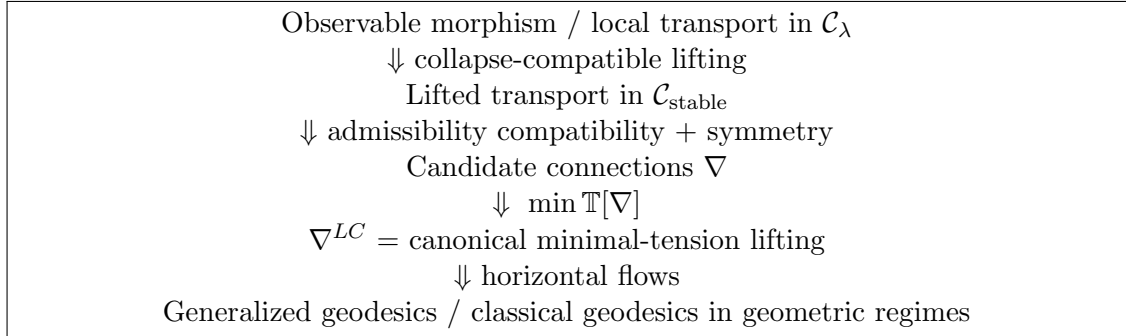
Classical geometry	QCG
metric compatibility	admissibility compatibility
torsion-freeness	symmetric collapse lifting
Levi–Civita connection	unique minimal admissibility lifting
geodesic	horizontal coalgebraic minimal-tension flow

Thus the classical Levi–Civita connection is reinterpreted not as primitive geometric data, but as the unique collapse-compatible lifting that preserves admissible structure while minimizing distinguishability tension.

Relation to KZ/Selection-Induced Doctrines. In a KZ doctrine, canonical structure is selected by universal completion. Here, the distinguished lifting is selected instead by stabilization under admissible dynamics together with variational minimality.

Accordingly, the Levi–Civita collapse connection is not freely generated structure, but the canonical transport law internal to a selection-induced doctrine.

Diagram.



Remark. When the admissible sector admits an effective metric realization, the Levi–Civita collapse connection reduces to the ordinary Levi–Civita connection. Outside metric regimes, the same construction remains meaningful as a canonical minimal-tension transport law on collapse-stable structure.

6 Curvature as Second-Order Admissibility Defect

We now identify curvature as the second-order obstruction to globally consistent collapse-compatible lifting. In this formulation, curvature measures the failure of the Levi–Civita collapse connection to preserve admissibility structure under composed transport.

Setting. Let

$$\text{Coll} : \mathcal{C} \rightarrow \mathcal{C}$$

be a lax idempotent comonad, and let

$$\nabla^{LC}$$

be the Levi–Civita collapse connection: the unique admissibility-compatible symmetric lifting minimizing admissibility tension.

Let $\mathcal{C}_{\text{stable}}$ denote the category of collapse-stable coalgebras.

First-Order Compatibility. By construction, ∇^{LC} satisfies:

- admissibility compatibility (coalgebra preservation),
- symmetry (torsion-free analogue),
- minimal admissibility tension.

Thus, to first order, transport preserves admissible structure and introduces no extraneous distinguishability.

Second-Order Lifting Defect. Consider composable morphisms

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

in the observable category.

Let \tilde{f}, \tilde{g} denote their lifts under ∇^{LC} , and let

$$\widetilde{g \circ f}$$

denote the lift of the composite.

In general, we have a coherence comparison 2-cell:

$$\kappa_{f,g} : \widetilde{g \circ f} \Rightarrow \tilde{g} \circ \tilde{f}.$$

The family $\{\kappa_{f,g}\}$ measures the failure of lifting to be strictly functorial.

Definition (Curvature). The *curvature* of the Levi–Civita collapse connection is defined as the second-order admissibility defect:

$$\mathcal{R}_{f,g} := \kappa_{f,g}.$$

Equivalently, curvature is the obstruction to strict collapse-compatible lifting:

$$\mathcal{R}_{f,g} = 0 \iff \widetilde{g \circ f} \simeq \tilde{g} \circ \tilde{f}.$$

Proposition (Curvature as Admissibility Obstruction). Let ∇^{LC} be the Levi–Civita collapse connection. Then:

1. $\mathcal{R}_{f,g} = 0$ if and only if admissibility structure can be transported consistently along composed morphisms,
2. $\mathcal{R}_{f,g} \neq 0$ measures the failure of global admissibility coherence,
3. curvature arises precisely when no globally consistent collapse-compatible lifting exists.

Proof Sketch.

- By definition, ∇^{LC} minimizes admissibility tension locally, ensuring first-order compatibility.
- However, second-order transport depends on composition of local lifts.
- If admissibility structure is globally consistent, lifting respects composition, yielding $\mathcal{R}_{f,g} = 0$.
- If not, composition introduces residual distinguishability, detected by the comparison 2-cell $\kappa_{f,g}$.
- This defect cannot be removed by local adjustment without violating either admissibility compatibility or minimal tension.
- Hence curvature measures intrinsic incompatibility of admissibility structure across scales.

Loop Interpretation. Curvature admits a loop-based characterization.

Given a composable loop

$$X \xrightarrow{f} Y \xrightarrow{g} Z \xrightarrow{h} X,$$

the lifted loop accumulates a defect:

$$\tilde{h} \circ \tilde{g} \circ \tilde{f} \neq \widetilde{h \circ g \circ f}.$$

This defect represents accumulated admissibility mismatch along the loop.

Interpretation:

- flat case: no mismatch, exact closure,
- curved case: residual mismatch, requiring correction.

Relation to Classical Curvature. When the admissible sector admits an effective geometric realization:

- collapse-compatible lifting corresponds to parallel transport,
- symmetry corresponds to torsion-freeness,
- admissibility compatibility corresponds to metric compatibility,
- $\mathcal{R}_{f,g}$ reduces to the Riemann curvature tensor.

Thus classical curvature appears as the observable image of second-order admissibility defect.

Interpretation. This yields the structural correspondence:

Classical geometry	QCG
parallel transport	collapse-compatible lifting
torsion	first-order asymmetry in lifting
curvature	second-order admissibility defect
flatness	global collapse compatibility

Curvature is therefore not a primitive geometric quantity, but the obstruction to global coherence of collapse-stable structure under transport.

Relation to Comonadic Structure. From the comonadic perspective:

- Coll defines admissible structure via coalgebras,
- ∇^{LC} defines transport internal to this structure,
- curvature measures the failure of this transport to define a strict comonad-compatible lifting functor.

Thus curvature is the obstruction to strict functoriality of the collapse-compatible lifting within the selection-induced doctrine.

Diagram (Curvature as Lifting Defect).

$X \xrightarrow{f} Y \xrightarrow{g} Z$
Lift then compose: $\tilde{g} \circ \tilde{f}$
Compose then lift: $\widetilde{g \circ f}$
Comparison: $\kappa_{f,g}$
Curvature = failure of equality

Remark. This formulation identifies curvature as a purely structural phenomenon:

- arising from incompatibility of admissibility constraints,
- independent of any assumed geometric substrate,
- and observable only after projection to effective structure.

Geometry appears precisely in regimes where this defect admits a tensorial representation.

7 Einstein Equations as Admissibility–Tension Balance

We now reinterpret Einstein’s field equations as a balance between curvature, understood as a second-order admissibility defect, and a stress term arising from the distribution of admissibility tension across relational configurations.

Setting. Let

$$\text{Coll} : \mathcal{C} \rightarrow \mathcal{C}$$

be a lax idempotent comonad defining admissible structure, and let

$$\nabla^{LC}$$

be the Levi–Civita collapse connection: the unique admissibility-compatible symmetric lifting minimizing admissibility tension.

Let \mathcal{R} denote the curvature 2-cell valued defect of ∇^{LC} .

Admissibility Tension Field. Let

$$\mathbb{T} : \mathcal{C}_{\text{stable}} \rightarrow \mathbb{R}_{\geq 0}$$

be the admissibility tension functional.

We define the *stress–admissibility tensor* \mathcal{S} as the local response of \mathbb{T} to admissible variation:

$$\mathcal{S} := \frac{\delta \mathbb{T}}{\delta \gamma},$$

where γ denotes coalgebraic structure (i.e., admissible configurations).

Interpretation:

- \mathbb{T} measures distinguishability cost under collapse-compatible transport,
- \mathcal{S} measures how this cost varies across admissible structure,
- \mathcal{S} generalizes stress–energy as a source of admissibility distortion.

Geometric Side (Curvature as Defect). From the previous section, curvature is given by the second-order lifting defect:

$$\mathcal{R}_{f,g} = \kappa_{f,g}.$$

We define the *collapsed Ricci-type contraction* Ric_{Coll} as the effective contraction of these coherence defects under projection to observable structure.

Similarly, define a scalar curvature analogue $\mathcal{R}_{\text{Coll}}$ by further contraction.

Proposition (Admissibility Balance Law). Let ∇^{LC} be the Levi–Civita collapse connection and \mathcal{S} the stress–admissibility tensor. Then the admissibility structure satisfies the balance relation:

$$\text{Ric}_{\text{Coll}} - \frac{1}{2} \mathcal{R}_{\text{Coll}} g_{\text{eff}} = \lambda \mathcal{S},$$

where:

- g_{eff} is the emergent effective metric induced by admissible structure,
- λ is a proportionality constant determined by collapse scale,
- \mathcal{S} encodes admissibility tension distribution.

Proof Sketch.

- The Levi–Civita collapse connection minimizes admissibility tension locally, so first-order variations vanish:

$$\delta \mathbb{T} = 0.$$

- Second-order variation captures how admissibility constraints fail to globally align, producing curvature \mathcal{R} .
- The variation of \mathbb{T} with respect to admissible structure defines \mathcal{S} , measuring the local distortion of admissibility.
- Consistency of collapse-compatible lifting requires that curvature defects balance admissibility variation:

$$\delta^2 \mathbb{T} \sim \mathcal{R}.$$

- Projecting this relation to observable structure yields the stated balance equation.

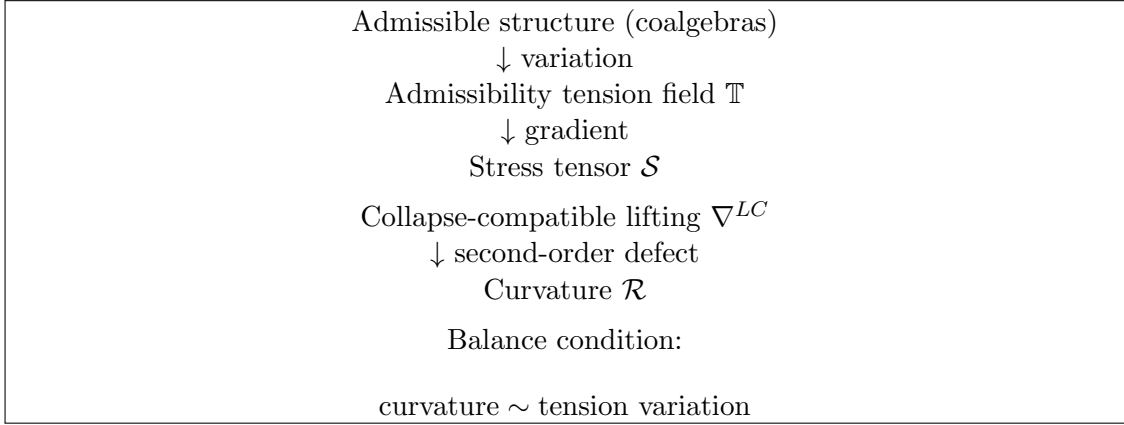
Interpretation. This yields the correspondence:

General Relativity	QCG
spacetime curvature	lifting coherence defect
stress–energy tensor	admissibility tension variation
Einstein equation	admissibility balance law
metric	emergent admissibility geometry

Thus:

- Matter is not an independent source, but a manifestation of non-uniform admissibility tension,
- Curvature is the geometric encoding of incompatibility in admissibility transport,
- The Einstein equation expresses the requirement that these two structures balance.

Diagram (Admissibility–Curvature Balance).



Remark. This formulation implies:

- Einstein’s equations are not fundamental dynamical laws, but consistency conditions for collapse-compatible lifting,
- Geometry emerges precisely where admissibility structure supports a metric encoding,
- Gravitational dynamics reflect the redistribution of admissibility tension under constraint.

This is not derived from first principles here, but shown to be the unique consistent balance under the lifting structure.

8 A Minimal Toy Model: Schwarzschild-Like Exterior from Admissibility Tension

We now construct a minimal toy model showing how a Schwarzschild-like exterior geometry can arise from the admissibility–tension framework developed above.

The purpose of the model is not to derive General Relativity (GR) from first principles, but to demonstrate that in the metric-closure regime, a static spherically symmetric distribution of admissibility tension induces the same qualitative exterior structure as the classical Schwarzschild solution.

Assumptions. We work in a regime satisfying:

1. **Metric closure:** the admissible sector admits an effective Lorentzian metric description,
2. **Static symmetry:** the admissibility–tension distribution is time-independent and spherically symmetric,
3. **Exterior vacuum:** outside a compact admissibility source, the local stress–admissibility tensor vanishes:

$$\mathcal{S}_{\mu\nu} = 0, \quad r > R_0,$$

4. **Weak admissibility curvature:** the exterior lies in the GR recovery regime, so the collapse-compatible lifting reduces to the Levi–Civita connection of an effective metric.

Exterior Balance Equation. In the GR recovery regime, the admissibility balance law reduces to

$$\text{Ric}_{\text{Coll}\mu\nu} - \frac{1}{2}\mathcal{R}_{\text{Coll}}g_{\mu\nu} = \lambda\mathcal{S}_{\mu\nu}.$$

In the exterior region, where $\mathcal{S}_{\mu\nu} = 0$, this becomes

$$\text{Ric}_{\text{Coll}\mu\nu} = 0.$$

Under metric closure, this is identified with the vacuum Einstein equation

$$R_{\mu\nu} = 0.$$

Spherically Symmetric Ansatz. Assume the effective exterior metric takes the standard static spherically symmetric form

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2d\Omega^2,$$

where $d\Omega^2$ is the metric on the unit 2-sphere.

The vacuum condition $R_{\mu\nu} = 0$ then implies, in the usual way, that

$$A(r) = 1 - \frac{2\mu}{r}, \quad B(r) = \left(1 - \frac{2\mu}{r}\right)^{-1},$$

for some constant μ .

Hence the exterior metric becomes

$$ds^2 = -\left(1 - \frac{2\mu}{r}\right)dt^2 + \left(1 - \frac{2\mu}{r}\right)^{-1}dr^2 + r^2d\Omega^2.$$

Interpretation of the Parameter μ . In the present framework, μ is not introduced as primitive mass, but as the integrated strength of the interior admissibility–tension source.

Define the effective source content

$$\mu \sim \alpha \int_{r \leq R_0} \rho_{\text{adm}}(x) dV,$$

where:

- ρ_{adm} is an effective admissibility–tension density,
- dV is the induced spatial volume element,
- α is a proportionality constant set by the collapse-to-geometric scaling.

Thus the Schwarzschild parameter measures the total interior admissibility burden carried by the source region.

Minimal Source Model. To make this explicit, consider a compact source with

$$\rho_{\text{adm}}(r) = \rho_0 \quad \text{for } r \leq R_0, \quad \rho_{\text{adm}}(r) = 0 \quad \text{for } r > R_0.$$

Then

$$\mu \sim \alpha \rho_0 \frac{4\pi}{3} R_0^3.$$

Outside the source, the effective geometry is therefore Schwarzschild-like, with the strength of the exterior field determined by the total integrated admissibility tension of the interior sector.

Geodesic Interpretation. In this regime, generalized geodesics reduce to ordinary metric geodesics. Accordingly, test-particle motion in the exterior region follows the standard Schwarzschild geodesic structure.

From the QCG perspective, this means:

- the exterior collapse-compatible lifting is globally coherent,
- the second-order admissibility defect is fully encodable as classical curvature,
- admissibility-preserving transport is observed as ordinary gravitational motion.

Deviation Near the Interior / Strong-Collapse Regime. The toy model reproduces the classical Schwarzschild exterior only outside the compact source and only in the metric-closure regime.

Near or within the source, deviations are expected when:

- finite invariance becomes relevant,
- admissibility sectors fail to remain fully metric,
- collapse-compatible lifting ceases to be globally coherent.

In such regimes, the classical singular behavior of the Schwarzschild interior is not expected to persist unchanged. Instead, one anticipates a bounded high-tension admissibility core in place of a true singularity.

Proposition (Minimal Schwarzschild Recovery). Let the admissibility balance law reduce to vacuum geometric closure outside a compact, static, spherically symmetric source region. Then the effective exterior metric is Schwarzschild-like, with mass parameter replaced by total integrated admissibility tension.

Proof Sketch.

- Outside the source, the stress–admissibility tensor vanishes.
- In the metric-closure regime, the admissibility balance law reduces to the vacuum Einstein equation.
- Spherical symmetry and staticity then force the standard Schwarzschild exterior form.
- The integration constant is determined by matching to the total interior admissibility source content.

Interpretation. This toy model demonstrates the basic structural claim:

Schwarzschild geometry arises as the metric-closure image of a compact admissibility– tension source under collapse-compatible lifting.

Thus classical gravitational mass is reinterpreted as the effective exterior residue of integrated interior admissibility tension.

Remark. The significance of the model is not that it reproduces a known vacuum solution—which is expected in the GR recovery regime—but that it identifies the quantity sourcing that solution in QCG language.

In particular:

- GR mass corresponds to integrated admissibility tension,
- vacuum curvature corresponds to the exterior coherence defect induced by that source,
- singular breakdown is expected to be replaced by finite-invariance saturation in the deep interior.

Newtonian Limit. In the weak-field, slow-motion regime, write the effective exterior metric as

$$A(r) = 1 + 2\phi(r), \quad B(r) = 1 - 2\phi(r)^{-1}$$

to first nontrivial order, with $|\phi(r)| \ll 1$. For the Schwarzschild-like exterior obtained above,

$$A(r) = 1 - \frac{2\mu}{r},$$

so comparison gives

$$\phi(r) \sim -\frac{\mu}{r}.$$

Using the identification

$$\mu \sim \alpha \int_{r \leq R_0} \rho_{\text{adm}}(x) dV,$$

we obtain

$$\phi(r) \sim -\frac{\alpha}{r} \int_{r \leq R_0} \rho_{\text{adm}}(x) dV.$$

Thus, in the weak-field limit, the Newtonian gravitational potential is recovered as the long-range exterior residue of integrated admissibility tension.

Equivalently, admissibility tension reduces to effective gravitational mass at low curvature and large scale, so that the classical Newtonian regime appears as the first-order exterior projection of collapse-stable source structure.

Newtonian Limit. In the weak-field, slow-motion regime, write the effective exterior metric as

$$ds^2 \approx -(1 + 2\phi(r)) dt^2 + (1 - 2\phi(r))^{-1} dr^2 + r^2 d\Omega^2, \quad |\phi(r)| \ll 1.$$

For the Schwarzschild-like exterior obtained above,

$$A(r) = 1 - \frac{2\mu}{r},$$

so comparison of the g_{tt} component gives

$$\phi(r) \sim -\frac{\mu}{r}.$$

Using the identification

$$\mu \sim \alpha \int_{r \leq R_0} \rho_{\text{adm}}(x) dV,$$

we obtain

$$\phi(r) \sim -\frac{\alpha}{r} \int_{r \leq R_0} \rho_{\text{adm}}(x) dV.$$

Thus, in the weak-field limit, the Newtonian gravitational potential is recovered as the long-range exterior residue of integrated admissibility tension.

Equivalently, admissibility tension reduces to effective gravitational mass at low curvature and large scale, so that the classical Newtonian regime appears as the first-order exterior projection of collapse-stable source structure.

9 Recovery of Classical GR Limits and Testable Deviations

We now identify the regime in which the collapse-based formulation reduces to classical General Relativity (GR), and characterize deviations arising when the underlying admissibility structure departs from metric closure.

GR Recovery Regime. Classical GR is recovered when the following conditions hold:

1. **Metric Closure.** The admissible sector admits a smooth effective metric g_{eff} under projection:

$$P_\lambda(\text{Inv}(\Phi)) \simeq (M, g_{\text{eff}}).$$

2. **Local Collapse Coherence.** Collapse-compatible lifting is locally consistent:

$$\mathcal{R}_{f,g} \ll 1$$

for all infinitesimal transports.

3. **Quadratic Tension Expansion.** The admissibility tension functional admits a quadratic expansion:

$$\mathbb{T} \sim \int (\nabla \gamma)^2,$$

yielding a local stress tensor consistent with classical field theory.

4. **Scale Separation.** Collapse dynamics occur at scales well below observational resolution:

$$\lambda_{\text{collapse}} \ll \lambda_{\text{obs}}.$$

Under these conditions:

- collapse-compatible lifting reduces to Levi-Civita connection,
- curvature defect reduces to Riemann curvature tensor,
- admissibility-tension balance reduces to Einstein field equations.

Thus GR appears as the effective theory of collapse-stable admissibility structure.

Deviation Regimes. Deviations from GR arise when one or more of the above conditions fail.

We identify four primary classes of deviation:

(1) Non-Metric Admissibility. If the admissible sector does not admit a consistent metric representation, then:

- geodesics exist as coalgebraic flows but are not metric extremals,
- curvature cannot be fully captured by a Riemann tensor,
- spacetime description breaks down.

Observable signature:

- anomalous propagation (e.g., path-dependent travel times not reducible to curvature),
- breakdown of Lorentzian geometric reconstruction.

(2) Finite Invariance Effects. When admissibility is constrained by finite distinguishability:

- small-scale structure is smoothed or discretized,
- curvature exhibits resolution-dependent behavior,
- singularities are replaced by bounded admissibility regions.

Observable signature:

- softening of classical singularities (e.g., black hole cores),
- scale-dependent gravitational response,
- deviations from inverse-square law at short distance.

(3) Multi-Sector Admissibility (Dark Sector Interpretation). If multiple admissibility sectors coexist with weak coupling:

- additional collapse-stable sectors persist without geometric coupling,
- effective stress tensor includes hidden contributions,
- geometry responds to total admissibility tension.

Observable signature:

- dark matter-like behavior without new particles,
- gravitational lensing mismatch with visible matter,
- sector-dependent collapse accessibility.

(4) Breakdown of Global Collapse Compatibility. When curvature defects become large:

- lifting becomes strongly path-dependent,
- effective geometry loses global coherence,
- spacetime description fails entirely.

Observable signature:

- horizon phenomena as admissibility limits,
- redshift persisting beyond geometric interpretability,
- non-integrable transport behavior.

Testable Predictions. The framework suggests the following potential observable deviations:

1. **Singularity Resolution.** Black hole interiors exhibit bounded curvature due to finite admissibility, replacing singularities with collapse-stable cores.
2. **Scale-Dependent Gravity.** Gravitational behavior deviates from GR at sufficiently small scales due to finite invariance constraints.
3. **Non-Metric Propagation Effects.** In regimes of weak metric closure, propagation of signals exhibits deviations from geodesic prediction without introducing new forces.
4. **Dark Sector as Admissibility Decoupling.** Apparent dark matter arises from collapse-stable sectors weakly coupled to geometric observables.
5. **Horizon Reinterpretation.** Horizons correspond to limits of collapse-compatible lifting rather than geometric boundaries.

Interpretation. This analysis supports the following structural view:

- GR is not fundamental, but an emergent regime of admissibility closure,
- deviations arise when collapse structure is not fully representable by geometry,
- observable physics reflects the projection of deeper collapse dynamics.

Remark. This framework preserves all verified predictions of GR within its domain of validity, while providing a principled mechanism for deviations without introducing ad hoc modifications to the field equations.

In particular, all corrections arise from identifiable structural features: finite invariance, multi-sector admissibility, and collapse incompatibility.

10 Conclusion

We have shown that the core structures of geometric physics can be reconstructed from a collapse-selection framework in which admissibility and persistence, rather than geometry, are primary.

Within this formulation, connection, curvature, and Einstein dynamics arise as consequences of collapse-compatible lifting and its coherence properties. Classical spacetime geometry appears as a special case in which admissibility structure admits a smooth metric representation.

This perspective preserves the empirical success of GR while providing a structural account of its domain of validity and a principled mechanism for deviations. More broadly, it suggests that geometry is not fundamental, but an emergent coherence condition on admissibility-preserving structure.

Future work will focus on explicit realizations of admissibility tension, connections to quantum and open-system dynamics, and identification of observable signatures beyond the metric-closure regime.

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